A mathematical model for the description of biomagnetic fluid flow exposed to a magnetic field that accounts for both electric and magnetic properties of the biofluid is presented. This is achieved by adding the Lorentz and magnetization forces in the Navier-Stokes equations. To demonstrate the effects of magnetic fields, we consider the case of laminar, incompressible, viscous, the steady flow of a Newtonian biomagnetic fluid (i) between two parallel plates; and (ii) through a straight rigid tube with a 60% in diameter, 84% on area, axisymmetric stenosis. Two external magnetic fields were investigated: one produced by an infinite wire carrying constant current, and a dipole-like field. We show, numerically and analytically, that the wire produces an irrotational force that, independent of its intensity, only alters the pressure leaving the velocity field unaffected. In contrast, when the fluid is exposed to the dipole-like field, which generates a rotational force, then both pressure and velocity can be strongly influenced even at moderate field strengths. Similar trends were obtained when a time varying flow is simulated through the axisymmetric stenosis in the presence of the dipole-like rotational magnetic field. It is expected that our findings could have important applications in blood flow control. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: biofluid; Lorentz and magnetization forces; irrotational force; magnetic pressure; continuous/discontinuous Galerkin

1. INTRODUCTION

Investigation of flow response in the presence of a magnetic field is a promising area that has attracted significant attention in recent years. The idea that external magnetic fields can interact with biofluids and alter their behavior is relatively new and very promising. The result was the emergence of biomagnetic fluid dynamics (BFD, Haik et al. [1]) with abundant applications in biology, bioengineering, and the medical sciences. The most common biofluid is blood, which possesses some very interesting properties due to the presence of iron oxide in the hemoglobin molecule of red blood cells. Blood has been found to behave as a paramagnetic material in its deoxygenated state, whereas oxygenated blood behaves as a diamagnetic material [2]. Therefore, blood can be considered as a magnetic material and a BFD model describing it, is analogous to the fluid description of ferrofluids [3–7].

The BFD model introduced by Haik et al. [1] describes the effects of magnetic field gradients on blood. This interaction generates a magnetization force that vanishes if the magnetic field is constant. The model does not take into account any induced currents that can be generated in biofluids and...
considers them electrically poor conductors. However, experiments have shown that blood exhibits considerable dielectric properties. Specifically, Jaspard et al. [8] performed \textit{ex vivo} measurements of the electric conductivity of cow and sheep blood. They concluded that the conductivity increases significantly when the haematocrit decreases. Haik’s model has been adopted by Papadopoulos and Tzirtzilakis [9] who studied laminar, incompressible flow in a curved square duct under the influence of an applied magnetic field generated by a wire bearing a constant current, placed parallel to the longitudinal axis outside the duct. It was shown that the axial velocity and the secondary flow at the transverse plane were influenced considerably by the external magnetic field. Similar findings were presented in later studies by Tzirtzilakis [10] in an axisymmetric stenosis flow channel and by Kenjeres [11] in a realistic geometry arterial stenosis model.

The conductivity of the biofluid on the other hand, gives rise to a Lorentz force even in the case of constant magnetic fields [12]. If, in addition, the magnetization and polarization properties are neglected, the corresponding mathematical representation of the biofluid model is analogous to the well-known formulation of magnetohydrodynamics [12–14]. The resulting Lorentz force can be used, for example, to diverge the blood flow as it has been shown in [15], where a uniform magnetic field of magnitude larger than 10 T gave rise to induced voltages and currents, and reduced the blood flow by almost 10%. A similar result of reduced blood flow was obtained by Haik \textit{et al.} in [16] where a 30% flow decrease was experimentally measured in the presence of strong magnetic fields. The decreased flow rate was attributed solely to the apparent additive viscosity of blood caused by the magnetic field as the authors state that they have taken care in their experimental setup (positioning of tube in the magnet) to ensure any effects from the magnetic field gradients would cancel out.

The aim of this study is to determine the effects of the spatial gradients of the magnetic field on the flow field of a biofluid and also examine how these effects differ when rotational or irrotational magnetic fields are considered. Towards this, the model introduced by Tzirtzilakis [17], which includes both Lorentz and magnetization forces, is adopted, and the biofluid was modelled as a Newtonian fluid. The concept of controlling the flow of blood can have numerous applications such as the reduction of blood loss during surgery and the development of magnetic devices for cell separation to targeted delivery of drugs. The latter offers a very promising method to achieve increased drug concentration at the targeted sites with significantly reduced exposure of healthy tissue to the drug, thus increasing its effectiveness and reducing the required dosage.

The paper is organized as follows: Section 2 establishes the mathematical formulation describing the flow of a biofluid in the presence of magnetic fields for two different models: a two-dimensional laminar flow between parallel plates and a three-dimensional one through a straight rigid tube with a 60% in diameter, 84% on area, axisymmetric stenosis, of an incompressible and Newtonian biofluid. Two magnetic fields are considered: one generated by an infinite straight conducting wire bearing constant current and a second, dipole-like, rotational magnetic field. Section 3 describes the numerical implementation used for the solution of the Navier-Stokes equations. Section 4 presents the results and comparisons with exact solutions when these are available. Couette flow, Hartmann flow (HF), and flow between parallel plates in the presence of the magnetic field produced by the wire are considered. The first two cases are implemented as solver validation test cases and the third case shows the effects of irrotational forces on the flow. Results of steady and pulsatile stenotic flows under the influence of both rotational and irrotational magnetic fields are also presented. Section 5 contains a summary and conclusions.

\section{2. MATHEMATICAL FORMULATION}

\subsection{2.1. General setting}

The Navier-Stokes equations for an incompressible, homogeneous, and Newtonian fluid are the following:

\begin{equation}
\nabla \cdot \mathbf{v} = 0, \\
\rho \frac{D\mathbf{v}}{Dt} = \mu \nabla^2 \mathbf{v} - \nabla p + \mathbf{f},
\end{equation}

where \( D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla \) is the material derivative, \( \mathbf{v} \equiv (u, v, w) \) is the velocity vector, \( \mu \) and \( \rho \) are the (constant) dynamic viscosity and density, respectively, and \( \mathbf{f} \) is the body force per unit volume. For the case of biofluids (such as blood), the explicit form of the force term depends on the model considered. For the most general case of electromagnetic fields, full coupling of the Navier-Stokes and Maxwell equations must be carried out [18]. Once the time variation of the electric field in Ampere’s law is ignored and Ohm’s law is used to eliminate the electric field, an induction equation for the magnetic field can be derived. The induction equation can be coupled with the Navier-Stokes equations in the magnetohydrodynamic (MHD) approximation [19, 20]. In this work, the equations that describe the flow of an electrically conducting and magnetic biofluid under the influence of a time independent magnetic field is considered and the combined subset of Navier-Stokes and Maxwell equations is as follows.

\[
\nabla \cdot \mathbf{v} = 0, \\
\rho \frac{D\mathbf{v}}{Dt} = \mu \nabla^2 \mathbf{v} - \nabla p + \mathbf{J} \times \mathbf{B} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H}.
\]

In Equation (2), the time derivative of the magnetic field has been neglected as in the MHD approximation, the current density, \( \mathbf{J} \), is obtained from Ohm’s law and the magnetic flux density obeys the divergence free condition,

\[
\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \\
\nabla \cdot \mathbf{B} = 0,
\]

where \( \mathbf{B}, \mathbf{H}, \mathbf{M}, \mathbf{E}, \) and \( \mathbf{J} \) are the magnetic flux density, magnetic field intensity, magnetization, electric field, and current density, respectively. In the present study, the biofluid is considered to be charged neutral with no current component due to moving charged particles within the flow, yielding Equation (3) for the current density. \( \mu_0 \) is the magnetic permeability of vacuum and \( \sigma \) is the electric conductivity of the biofluid. Assuming that biofluids are poor electric conductors and neglecting gravity, the force term depends solely on the magnetization of the fluid and is expressed as a gradient of the external magnetic field. As a result, the Lorentz force, \( \mathbf{J} \times \mathbf{B} \), can be neglected and only the magnetization force, \( \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} \), is included in the right-hand side (RHS) of the momentum equation. On the contrary, if the biofluid exhibits considerable electric conductivity, both the magnetization and Lorentz forces must be taken into account. In the present study, the latter case is considered.

The relative strength of the two forces depends on the magnitudes of the electric conductivity and magnetization of the biofluid. For blood, the electric conductivity is strongly related to the red blood cells concentration, with a typical value for the case of stationary blood of \( \sigma = 0.7 \) S/m. For moving blood though the conductivity is increased due to the reorientation of red blood cells caused by viscous forces. It has been experimentally determined [21] that the value of \( \sigma \) for flowing blood is increased by approximately 20\% yielding, \( 0.7 \leq \sigma \leq 0.9 \) S/m.

The degree of magnetization of a material in the presence of external magnetic fields generally depends on its temperature and density, and the intensity of the magnetic field. As a result, various models exist in the literature that relate these quantities. The most accurate expression for the magnitude of the magnetization of red blood cells is given by [22, 23],

\[
M = Nm \left[ \coth \left( \frac{\mu_0 m H}{kT} \right) - \frac{kT}{\mu_0 m H} \right],
\]

where \( N, m, k, \) and \( T \) are the number of particles per unit volume, magnetic moment of a particle, Boltzmann constant, and absolute temperature, respectively. The following simpler expression is also valid for isothermal cases,

\[
M = \chi H,
\]

where \( \chi \) is the magnetic susceptibility, referred to also as volume magnetic susceptibility. For the case of blood, the magnetic susceptibility depends on its condition. Oxygenated blood is characterized by \( \chi^{\text{oxyg}} = -6.6 \cdot 10^{-7} \) (close to the magnetic susceptibility of water \( -7.19 \cdot 10^{-7} \)) whereas
for deoxygenated blood $\chi_{\text{deoxygen}} = 3.5 \cdot 10^{-6}$ [2]. In the present study, the biofluid considered mimics the properties of blood. As temperature variations in blood flow in vivo have not been found to influence the magnetization significantly [24], we assume the fluid in our model is isothermal and the expression in (4) has been used to model its magnetization.

2.2. The models considered

We next investigate the effects of external magnetic fields on the two models considered: the flow between two parallel plates and the flow through a straight rigid tube with a 60% axisymmetric stenosis. We assume that the flow is laminar and that the fluid is Newtonian and incompressible. Three different magnetic fields are implemented for this study. For validation purposes of the numerical technique, we consider a constant magnetic field in space. The second magnetic field is produced by an electrically conducting infinite straight wire of constant current, $I$, perpendicular to the $xy$-plane that points inwards and crosses the plane at $(x_i, y_i)$, yielding,

$$B(x, y) = K \left( \frac{y - y_i}{(x - x_i)^2 + (y - y_i)^2} \right) \mathbf{i} - K \left( \frac{x - x_i}{(x - x_i)^2 + (y - y_i)^2} \right) \mathbf{j},$$

where $K = \mu_0 I / 2\pi$. The magnitude, $|B(x, y)|$, equals,

$$|B(x, y)| = \frac{K}{\sqrt{(x - x_i)^2 + (y - y_i)^2}},$$

and the parameter $K$ determines the intensity of the field at any point. Lastly, a magnetic field whose components along the $x$, $y$, and $z$ directions are respectively given by,

$$B_x = -C \left( \frac{2(x - x_i)^2 - r^2}{r^6} \right),$$

$$B_y = -C \left( \frac{2(x - x_i)(y - y_i)}{r^6} \right),$$

$$B_z = -C \left( \frac{2(x - x_i)(z - z_i)}{r^6} \right),$$

is adopted, where $r = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$. The magnitude can be written as,

$$|B(x, y)| = \frac{C}{r^4},$$

and depends on the coordinates $x_i$, $y_i$, and $z_i$ of some arbitrary point and the parameter $C$. The interesting property of the latter with respect to the former is that not only it satisfies Gauss’s law but also produces a rotational magnetization force as is shown in Section 4. Additionally, it resembles the (irrotational) magnetic field of an ideal dipole, which can be applied in real patient treatment. In all cases, the forces generated due to the presence of the magnetic fields under consideration can be calculated using the relations,

$$\mathbf{J} = \sigma (\mathbf{v} \times \mathbf{B}),$$

$$\mathbf{H} = \frac{1}{\mu} \mathbf{B} = \frac{1}{\mu_0 (1 + \chi)} \mathbf{B},$$

$$\mathbf{M} = \chi \mathbf{H} = \frac{1}{\mu_0} \left( \frac{\chi}{1 + \chi} \right) \mathbf{B}.$$  

After some straightforward algebra, the components of the Lorentz and magnetization forces for the magnetic field (5) and the components of the magnetization force for the magnetic field (7) are summarized in Tables (I) and (II), respectively. It should be noted that the parameter $\alpha$ in both tables is given by,

$$\alpha = \frac{1}{\mu_0 \chi (1 + \chi)^2}. $$
and rigid plates at a fixed distance apart that lie on the 
`size of the plates in the x
pressure will be a function of u
takes place in the presence of a constant magnetic field,
and the flow is characterized by a parabolic profile. An extension of the plane Couette flow (PCF)
Defining the y
magnetic field is parallel to the x plates with a constant magnetic field,
B0 with magnitude,
the induced current, J
2.2.1. Model I: flow between parallel plates
Plane Couette flow: Let us consider the case of a channel consisting of two infinite, flat, parallel, and rigid plates at a fixed distance apart that lie on the y = a and y = −a planes. Assuming that the size of the plates in the z direction is infinite with respect to their separation distance, L = 2a, the pressure will be a function of x only, and there is no flow component in the y direction. The mean velocity distribution, um, is then proportional to the pressure drop,
and the flow is characterized by a parabolic profile. An extension of the plane Couette flow (PCF) takes place in the presence of a constant magnetic field, B0, normal to the plates. This is the so-called HF, which is a well-known application of MHD with an analytical solution.
Hartmann flow: Hartmann flows are defined as the class of one-dimensional flows between two plates with a constant magnetic field, B0, normal to them. For the case considered, the constant magnetic field is parallel to the y axis, thus affecting the flow along the x axis only. If no electric field is imposed on the x or y direction, and assuming that the plates are perfect electric insulators, the induced current, J = uσB0 k, generates a Lorentz force per unit volume along the −x direction with magnitude,
|fL| = |J × B| = uσB2 0.
(11)
Defining the Hartmann number, H, as,
H ≡ aB0 \sqrt{\frac{\sigma}{\mu}}.

Table I. Force components for an external magnetic field defined by Equation (5).

<table>
<thead>
<tr>
<th>Lorentz force (N/kg)</th>
<th>Magnitization force (N/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x  (f_{Lx} = -\frac{\sigma K^2 (x-x_i) [u(x-x_i) + v(y-y_i)]}{\rho (x-x_i)^2 + (y-y_i)^2} )</td>
<td></td>
</tr>
<tr>
<td>y  (f_{Ly} = -\frac{\sigma K^2 (y-y_i) [u(x-x_i) + v(y-y_i)]}{\rho (x-x_i)^2 + (y-y_i)^2} )</td>
<td></td>
</tr>
<tr>
<td>z  (f_{Lz} = -\frac{\sigma K^2 (x-x_i)^2 + (y-y_i)^2}{w} )</td>
<td></td>
</tr>
<tr>
<td>x  (f_{Mx} = -\frac{\alpha K^2}{\rho} \frac{(x-x_i)}{(x-x_i)^2 + (y-y_i)^2} )</td>
<td></td>
</tr>
<tr>
<td>y  (f_{My} = -\frac{\alpha K^2}{\rho} \frac{(y-y_i)}{(x-x_i)^2 + (y-y_i)^2} )</td>
<td></td>
</tr>
<tr>
<td>z  (f_{Mz} = 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Table II. Components of the magnetization force for an external magnetic field defined by Equation (7).

<table>
<thead>
<tr>
<th>Magnetization force (N/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x  (f_{Mx} = -\frac{4\alpha C^2}{\rho} \frac{(x-x_i)^3}{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} )</td>
</tr>
<tr>
<td>y  (f_{My} = -\frac{2\alpha C^2}{\rho} \frac{(y-y_i) 3(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} )</td>
</tr>
<tr>
<td>z  (f_{Mz} = -\frac{2\alpha C^2}{\rho} \frac{(z-z_i) 3(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} )</td>
</tr>
</tbody>
</table>

As expected, the component of the magnetization force along the z direction equals zero in Table (I) as the magnetic field (5) varies along the xy-plane only. However, for both models considered, the presence of the external magnetic field produces an asymmetric stenotic flow and a three-dimensional solution is required.

2.2.1. Model I: flow between parallel plates

Plane Couette flow: Let us consider the case of a channel consisting of two infinite, flat, parallel, and rigid plates at a fixed distance apart that lie on the y = a and y = −a planes. Assuming that the size of the plates in the z direction is infinite with respect to their separation distance, L = 2a, the pressure will be a function of x only, and there is no flow component in the y direction. The mean velocity distribution, um, is then proportional to the pressure drop,

\[ u_m = \frac{a^2}{3\mu} \left( \frac{dp}{dx} \right), \]

and the flow is characterized by a parabolic profile. An extension of the plane Couette flow (PCF) takes place in the presence of a constant magnetic field, B0, normal to the plates. This is the so-called HF, which is a well-known application of MHD with an analytical solution.

Hartmann flow: Hartmann flows are defined as the class of one-dimensional flows between two plates with a constant magnetic field, B0, normal to them. For the case considered, the constant magnetic field is parallel to the y axis, thus affecting the flow along the x axis only. If no electric field is imposed on the x or y direction, and assuming that the plates are perfect electric insulators, the induced current, J = uσB0 k, generates a Lorentz force per unit volume along the −x direction with magnitude,

\[ |f_L| = |J \times B| = u\sigma B^2_0. \] (11)

Defining the Hartmann number, H, as,

\[ H \equiv aB_0 \sqrt{\frac{\sigma}{\mu}}. \]
the velocity distribution is then given by,

\[ u(y) = -\frac{1}{\sigma B_0} \left( \frac{dp}{dx} \right) \left[ 1 - \frac{\cosh(Hy/a)}{\cosh(H)} \right]. \tag{12} \]

The profile therefore in the fully developed region of the flow depends on the value of the Hartmann number as expected. For large Hartmann numbers (high intensity magnetic fields), the velocity profile gradually becomes flatter, with steeper velocity gradients near the solid walls in order to satisfy the no-slip boundary condition.

**Plane Couette flow with magnetization force:** The third and last case of flow between two parallel plates considered in this study is the PCF in the presence of an external magnetic field defined by Equation (5), where \((x_i/L, y_i/L) = (3, -1)\). Assuming that the plates are located at \(y/L = \pm 0.5\), and noticing that the closest point of the \(y = -\alpha\) plate to the wire has coordinates \((x_0/L, y_0/L) = (3, -0.5)\), it can easily be shown that,

\[ |B(x, y)|_{\text{max}} = |B(x_0, y_0)| = 4 \text{ T}, \quad \text{for} \quad K = 0.02 \text{ Tm and } L = 0.01 \text{ m}, \tag{13} \]

as illustrated in Figure 1. A closed solution of the momentum equation cannot be obtained for this case because of the spatially varying magnetization force that appears as an extra term in the RHS of the momentum equation. The flow thus is solved, numerically utilizing the technique described in [25]. The computational domain extends three channel widths before and seven after the position the wire crosses the \(xy\)-plane (or equivalently the \(x/L = 3\) plane) in order to allow sufficient length for flow development.

**2.2.2. Model II: flow through stenosis.** We consider a straight rigid tube with a 60% (in diameter) axisymmetric stenosis (Figure 2). The computational domain extends three diameters upstream and 10 or 15 (for steady or pulsatile flows respectively) downstream of the peak stenosis in order to allow post-stenotic flow development. The geometry of the flow domain and the distribution of the magnetic field closely follow the one used by Tzirtzilakis [10]. The cross section of the stenosis with the \(xy\)-plane yields two curves: \(F_1(x)\), which is characterized by negative values \(\forall x \in [0, L_1 + L_2]\), and \(F_2(x)\) with corresponding positive values \(\forall x \in [0, L_1 + L_2]\). Both curves are defined by the following two equations,

\[ F_1(x) = A \text{sech}[B(x - x_0)] - D/2, \]

\[ F_2(x) = D/2 - A \text{sech}[B(x - x_0)], \tag{14} \]

![Figure 1](image1.png)  
**Figure 1.** Magnitude contours of the external magnetic field (5) with \(K = 0.02 \text{ Tm}, \text{yielding} \ |B(x_0, y_0)| = 4 \text{ T} \text{ at} \ (x_0/L, y_0/L) = (3, -0.5). \text{The magnetic field is generated by an infinite straight wire bearing constant current normal to the xy-plane at} \ (x_i/L, y_i/L) = (3, -1) \text{ with an inward direction.}

![Figure 2](image2.png)  
**Figure 2.** Straight rigid tube with a 60% axisymmetric stenosis. The characteristic lengths are as follows: \(L_1 = 3D\) and \(L_2 = 10D\) for steady flows (shown here) and \(L_2 = 15D\) for pulsatile flows.
where the constants $A$ and $B$ determine the constriction and extension of the stenosis, respectively. The parameter $x_0$ is the $x$ component of the maximum constriction defined as the global maximum of $F_1(x_0)$. If $A = 0$ (or $B = 0$), there is no stenosis, and the geometry defined by Equations (14) is a cylinder of diameter $D$ (or $D - 2A$) centered at the origin. Concerning the present analysis, the stenosis is parametrized using $A = 0.3D$, $B = 6/D$, $x_0 = 3D$, and $D = 0.01$ m. Three cases are considered: the first one is the simple configuration where no external magnetic field is applied (steady and pulsatile flows). In the second case, the magnetic field is defined by Equation (5) with $(x_i/D, y_i/D) = (3, -0.5)$ as it can be seen from Figure 3. The parameter $K = 0.01024$ Tm, so that $|B(x, y)|_{\max} = |B(x_0, y_0)| = 4$ T, where $(x_0/D, y_0/D) = (2.83, -0.31)$ are the coordinates of one of the two closest points of the stenosis to the wire (steady flow). In the third and last case, the magnetic field is defined by Equation (7). The coordinates $(x_i, y_i)$ are kept constant, with $C = 1.72 \cdot 10^{-10}$ Tm$^4$, yielding $|B(x, y)|_{\max} = |B(x_0, y_0)| = 4$ T at $(x_0/D, y_0/D) = (2.8, -0.3)$ as illustrated in Figure 4 (steady and pulsatile flows).

The components of the magnetization forces, $f_M$, for the two magnetic fields considered earlier are presented in Tables I and II. Under close inspection it can be seen that the magnetization force produced due to the presence of the infinite wire possesses an interesting property as opposed to the one generated by the dipole-like magnetic field. It is irrotational for any simply connected subset that excludes the wire, namely,

$$\nabla \times f_M = 0. \quad (15)$$

Consequently, it can be written as the gradient of some scalar quantity, which can be called the magnetic pressure, $p_m$,

$$f_M = \nabla p_m, \quad (16)$$

because,

$$\nabla \times f_M = \nabla \times \nabla p_m = 0, \ \forall p_m. \quad (17)$$

It can be shown that in our case, the explicit form of the magnetic pressure is given by,

$$p_m = \frac{\alpha}{2} \frac{K^2}{(x-x_i)^2 + (y-y_i)^2} = \frac{\alpha}{2} |B(x, y)|^2. \quad (18)$$

Figure 3. Magnitude contours of the external magnetic field (5) with $K = 0.01024$ Tm, yielding $|B(x_0, y_0)| = 4$ T at $(x_0/D, y_0/D) = (2.83, -0.31)$. The magnetic field is generated by an infinite straight wire bearing constant current normal to the $xy$-plane at $(x_i/D, y_i/D) = (3, -0.5)$ with an inward direction.

Figure 4. Magnitude contours of the external magnetic field (7) with $C = 1.72 \cdot 10^{-10}$ Tm$^4$, yielding $|B(x_0, y_0)| = 4$ T at $(x_0/D, y_0/D) = (2.8, -0.3)$. For visualization purposes the intensity of the field is plotted on a natural logarithmic scale.
The irrotationality property of the magnetization force as expressed by Equation (15) stems from the fact that the corresponding magnetic field is also irrotational, yielding,
\[ f_M = \mu_0 (M \cdot \nabla)H = \alpha (B \cdot \nabla)B = \frac{\alpha}{2} \nabla B^2 - \alpha B \times (\nabla \times B) = \nabla \left( \frac{\alpha}{2} |B(x, y)|^2 \right). \]  
(19)

As a result, the magnetization force acts in a similar way to pressure and the momentum equation becomes,
\[ \rho \frac{Dv}{Dt} = \mu \nabla^2 v - \nabla (p - p_m). \]  
(20)

The curl of Equation (20) eliminates both pressure and magnetic pressure. Therefore, the magnetization force produced by an infinite straight wire bearing constant current does not add any vorticity to the fluid or invalidate Kelvin’s theorem. A total (or modified) pressure, \( p_{\text{total}} \), can then be introduced as,
\[ p_{\text{total}} = p - p_m + c = p - \frac{\alpha}{2} |B(x, y)|^2 + c, \]  
(21)

where \( c = \text{constant} \), and Equation (20) takes the form,
\[ \rho \frac{Dv}{Dt} = \mu \nabla^2 v - \nabla p_{\text{total}}. \]  
(22)

Therefore, the gradient of pressure (or total pressure in this case) is an irrotational force. In the absence of free surfaces or stratification, irrotational forces do not alter the velocity distribution of an incompressible fluid regardless of the intensity of the magnetic field.\(^2\) The pressure balances any effects introduced by the magnetic pressure term, leaving the kinematic part of the flow totally unaffected. It should also be noted that care should be taken in modelling these types of problems to extend the outlet boundary at a distance away for the magnetic field source that ensures the magnetization force has completely diminished and thus will not invalidate the outlet boundary conditions.

3. NUMERICAL IMPLEMENTATION

The results were obtained using an open source, parallel, adaptive mesh refinement solver called gnuid, developed by Botti and Di Pietro [25]. It is based on a pressure-correction scheme for the incompressible Navier-Stokes equations, combining a discontinuous Galerkin (DG) approximation for the velocity and a continuous Galerkin (CG) approximation for the pressure, proposed by Guermond and Quartapelle [26]. It treats the momentum equation implicitly based on either a backward Euler method or a second order backward differentiation formula.

Time discretization can be summarized briefly as follows:\(^3\): consider a partition of an appropriate time domain, \((t_{in}, t_f)\), into equal intervals, \(\Delta t\). Without loss of generality, \(t_{in} = 0\), yielding the time at step \(n\), \(t_n \equiv n \Delta t\). Define next the sequence of triplets, \((\tilde{u}^{n+1}, u^{n+1}, p^{n+1})\), at step \(n + 1\) in such a way that \(u^{n+1}\) accounts for the convection and diffusion in the momentum equation, whereas \(\tilde{u}^{n+1}\) incorporates the incompressibility constraint. This is the essence of the projection method used in order to decouple the solution of the momentum equation from the divergence-free constraint, by simply decomposing the vector field into a divergence-free part and an irrotational part.

\(^2\)This is true only for magnetic fields that have no time dependence. Time dependent magnetic fields may produce irrotational magnetization forces that can alter the flow, even if the field itself is irrotational.

\(^3\)This section is a brief summary of the techniques presented in [25]. Refer to the paper for a detailed description of the method.
For any bounded and open set, $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, the pressure-correction scheme using a second order backward differentiation formula becomes,

$$
\left( \frac{\beta_0}{\Delta t} - \nu \Delta \right) u^{n+1} + \left( u^{n+1}_j \partial_j \right) u^{n+1} + \frac{1}{2} (\nabla \cdot u^{n+1}) u^{n+1}
= f^{n+1} - \left( \frac{\beta_1}{\Delta t} u^n + \frac{\beta_2}{\Delta t} u^{n-1} \right) - \nabla (\gamma_0 p^n + \gamma_1 p^{n-1} + \gamma_2 p^{n-2}) \text{ in } \Omega,
$$

(23)

where,

$$
u^{n+1} = 0 \text{ on } \partial \Omega.
$$

(24)

The parameters $\beta_i$ and $\gamma_i$, $i = 0, 1, 2$, equal, $\beta_0 = \frac{3}{2}$, $\beta_1 = -2$, $\beta_2 = \frac{1}{2}$, and $\gamma_0 = \frac{7}{3}$, $\gamma_1 = -\frac{5}{3}$, $\gamma_2 = \frac{1}{3}$ [27]. The projection step can then be written as,

$$
\Delta(p^{n+1} - p^n) = \frac{\beta_0}{\Delta t} \nabla \cdot u^{n+1} \text{ in } \Omega,
$$

$$
\partial_n(p^{n+1} - p^n) = 0 \text{ on } \partial \Omega.
$$

(25)

For the space discretization, it is important to introduce some notation first. Assume a subdivision, $\mathcal{T}$, of $\Omega$ into elements $T$. The union of all element faces, $F$, can then be collected into its interior part, $F$, defined as $F = F \cap \partial \Omega$ yielding $F = F \cup \partial \Omega$. By construction, interior elements will share faces among each other. Let $T^+$ and $T^-$ be two different elements sharing a face $F_f = \partial T^+ \cap \partial T^-$. For any function $\phi : \Omega \rightarrow \mathbb{R}^d$ then, we define the jump and the average of $\phi$ respectively as,

$$
[\phi] = \phi_{T^+} - \phi_{T^-}, \quad \text{and} \quad \{\phi\} = \frac{1}{2} (\phi_{T^+} + \phi_{T^-}).
$$

In the special case where $F_f \subset \partial T \cap \Omega$, the jump and average operators are set as $[\phi] = \{\phi\} = \phi_{T^+}$.

We next define the set of polynomials of order less or equal to $k$ for $d$ variables, $\mathbb{P}_d^k$. The velocity and pressure originate from the spaces $U_h$ and $P_h$,

$$
U_h \equiv [dG(k)]^d, \quad \text{and} \quad P_h \equiv c G(k)/\mathbb{R},
$$

where $k \geq 1$ and,

$$
dG(k) \equiv \left\{ v_h \in L^2(\Omega) \forall T \in \mathcal{T}, v_h \in \mathbb{P}_d^k(T) \right\},
$$

$$
cG(k) \equiv \left\{ q_h \in C^0(\tilde{\Omega}) \forall T \in \mathcal{T}, q_h \in \mathbb{P}_d^k(T) \right\}.
$$

Following the discretization of Arnold [28] for the diffusive term and Di Pietro and Ern [29] for the convective term of the momentum equation, the discretized form of Equation (23) for $u_h^{n+1} \in U_h$ is given by [25],

$$
\frac{\beta_0}{\Delta t} m_h(u_h^{n+1}, v_h) + \nu a_h(u_h^{n+1}, v_h) + t_h(u_h^n, u_h^{n+1}, v_h) + t_h(u_h^{n+1}, u_h^n, v_h)
= \int_\Omega f \cdot v_h - \frac{1}{\Delta t} m_h(u_h^n, v_h) + b_h(v_h, p_h^n) + t_h(u_h^n, u_h^n, v_h) \quad \forall v_h \in U_h,
$$

(26)

where

$$
\begin{align*}
 u_h^* &= \beta_1 u_h^n + \beta_2 u_h^{n-1}, \\
p_h^* &= \gamma_1 p_h^n + \gamma_2 p_h^{n-1} + \gamma_3 p_h^{n-2},
\end{align*}
$$

(27)

and $m_h(u_h^{n+1}, v_h)$ is the bilinear form associated to the time derivative discretization, $a_h(u_h^{n+1}, v_h)$ the bilinear form of the diffusive term in the momentum equation, and $t_h(w_h, u_h, v_h)$ the nonlinear...
convective term in the momentum equation [25]. Finally, the discretized form of Equation (25) for 
$p_h^{n+1} \in \mathcal{P}_h$ reads,

$$ \int_{\Omega} \nabla p_h^{n+1} \cdot \nabla q_h = - \frac{B_0}{\Delta t} b_h (u_h^{n+1}, q_h) + \int_{\Omega} \nabla p_h^n \cdot \nabla q_h, \quad \forall q_h \in \mathcal{P}_h,$$

(28)

where $b_h(v_h, q_h)$ is the discrete velocity divergence and pressure gradient.

The space discretization that was chosen for the needs of this analysis relies on a $DG(1) - CG(1)$
approximation for the steady flows and on a $DG(2) - CG(1)$ approximation for the pulsatile flows.
In both cases, the polynomial space $\mathcal{P}_h^k$ is monomials for the discontinuous space and Lagrange
polynomials for the continuous space. The results are presented in the following section.

4. RESULTS

The domain is discretized with linear hexahedral elements (approximately 2000 for the parallel
plates, 120 000 and 180 000 for the steady and pulsatile flows through the stenosis, respectively).
Whenever stretched meshes are used, a sufficient number of elements is clustered close to the wall
in order to capture the steep near-wall velocity gradients. The first layer of elements off the wall has
a distance of $5 \cdot 10^{-3} D$, where $D$ is the characteristic length. For steady flows, a fully developed
profile is prescribed at the inlet of the domain. In addition, the inlet of the stenosis is placed three
diameters away from the throat (which acts as a solid obstruction) in order to avoid generating non-
physical solutions. The pressure is not fixed at the inlet because it is adjusted to whatever value is
appropriate to the prescribed velocity distribution. The Reynolds number is set as follows: $Re = 300$
for the parallel plates flow and $Re = 100$ for the steady and pulsatile stenotic flows. In all cases, a
constant pressure boundary condition is imposed at the outlet. Because there is no flow reversal, the
specific choice of boundary conditions at the outflow of the domain is valid. The biofluid considered
mimics the rheological properties of blood with $\mu = 0.0035$ Pa $\cdot$ s, and $\rho = 1050$ kg/m$^3$ [30]. In
terms of its electromagnetic properties, a constant conductivity of $\sigma = 0.8$ S/m is selected, which
is similar to that of flowing blood, and the biofluid is considered to behave as deoxygenated blood
with $\chi = 3.5 \cdot 10^{-6}$. Additionally, the outlet of the stenosis is at least ten diameters away from
the magnetic field source. Thus, the field intensity, because of the distance from the source and the
steep gradients near the source (see Figures 3 and 4) has decreased significantly yielding negligible
gradient forces at the outlet of the stenosis.

4.1. Case I: flow between parallel plates

In this section, we numerically compute the flow between two parallel plates for three different cases
and compare it with analytical solutions when these are available. The first case consists of the simple
viscous flow in the absence of any external forces, the well known PCF. We then consider the
flow in the presence of a constant in space magnetic field, $B_0$, normal to them. This is the so-called
HF, an example of MHD flow with an exact solution. Lastly, we study the effects on the flow of the
irrotational magnetization force generated by the external magnetic field defined by Equation (5)
with $K = 0.02$ Tm for the PCF.

4.1.1. Comparison with exact solutions. The numerically calculated velocity distributions for the
PCF and HF are compared with the corresponding analytical solutions in Figure 5. The channel
width $L = 2a = 0.01$ m and the mean velocity at the inlet, $u_m = 0.1$ m/s in both cases. The numerical
solutions are practically identical to the theoretical profiles. For the HF, the relation between the
applied magnetic field and the Hartmann number takes the form $B_0 = 5 \sqrt{H}$. A choice of $H = 5$
therefore requires a quite high intensity constant magnetic field $B_0 = 25 \sqrt{7}$ T along the $y$ direction.

As pointed out in Section 2.2.2, irrotational forces do not alter the velocity distribution under cer-
tain conditions (absence of free surfaces, constant initial conditions). They will modify the pressure
distribution but because the absolute value of pressure is irrelevant for incompressible flow, we can
freely add the potential of any irrotational force without affecting the velocity field. For example, the
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Figure 5. Streamwise velocity for the plane Couette flow and Hartmann flow of a biofluid that mimics that rheological properties of blood, through a channel of width $L = 0.01$ m and mean velocity $u_m = 0.1$ m/s. The numerical results in both cases are in total agreement with the analytical ones.

Figure 6. Percentage deviation of the streamwise velocity at the $x/L = 3$ plane in the presence of a magnetic field defined by Equation (5) with $K = 0.02$ Tm for two different meshes. The deviation of the numerical result from the analytical one strongly depends on the size of the elements, and in both cases it is no more than a few tenths of one percent.

magnetization force due to the presence of an external magnetic field produced by an infinite straight wire carrying constant current is an irrotational force. It is the pressure then that must balance any effects produced, leaving the kinematic part unaffected.

Figures 6, 7, and 8 present the effects of the irrotational magnetization force produced by a magnetic field defined by Equation (5) with $K = 0.02$ Tm for the PCF. The distribution of the magnetic field magnitude on the $xy$-plane is shown in Figure 1. Figure 6 shows the deviation of the streamwise velocity in the presence of the magnetic field, $u_M$, with respect to its unperturbed counterpart, $u$, at the $x/L = 3$ plane. Two different meshes are used for the computation. One with clustered elements close to the wall and one with uniform mesh where all the elements are separated by a distance equal to $5 \cdot 10^{-3}L$. It is clear that the velocity field is not affected by the presence of the

---

1It should be noted though that the specific flow was also studied for $K = 0.25$ Tm, yielding $|B(x,y)|_{\text{max}} = |B(x_0,y_0)| = 50$ T. The results were the same qualitatively. The velocity profiles were not affected by the extreme magnetic field (and the resulting irrotational magnetization force), whose presence was manifested as a pressure change solely.
Figure 7. Pressure distribution one channel width upstream to four downstream the $x/L = 3$ plane, in the presence of a magnetic field defined by Equation (5) with $K = 0.02$ Tm. A: pressure of the unperturbed flow, B: pressure in the presence of the magnetization force, C: magnetic pressure, and D: total pressure as defined by Equation (21). The pressure distribution of the unperturbed flow and the total pressure are equal, leaving the velocity distribution unaffected. The wire is located at $(x/L, y/L) = (3, -1)$.

Figure 8. Pressure components at the $x/L = 3$ plane in the presence of a magnetic field defined by Equation (5) with $K = 0.02$ Tm. Pressure of the unperturbed flow (squares), pressure in the presence of the magnetization force (dash dot), magnetic pressure (dash), total pressure (circles). The total pressure equals the pressure of the unperturbed flow.

magnetization force, and that the error strongly depends on the size of the elements, although in both cases it is less than 0.3%.

Figures 7 and 8 demonstrate that the pressure in the presence of the magnetic field adjusts in order to accommodate for the existence of the magnetic pressure. The result is a total pressure that is exactly the same (or shifted by a constant factor to be exact) to the one obtained in the absence of the magnetic field.
Figure 9. Streamwise velocity one diameter upstream to four downstream of the stenosis, in the presence of a magnetic field defined by Equation (5) with $K = 0.01024 \text{Tm}$. A: unperturbed flow, B: perturbed flow due to the magnetization force, C: perturbed flow because of both forces. The wire is located at $(x/D, y/D) = (3, -0.5)$.

4.2. Case IIa: steady flow through stenosis

In this section, the results of the numerical simulation are presented for the steady flow through the straight rigid tube with a 60% axisymmetric stenosis considered in Section 2.2.2. All the stenosis cross sections are taken on the $z = 0$ plane. Additionally, all the plots present the values of the corresponding quantities along the axis of symmetry. Two different magnetic fields are considered in this section. An irrotational magnetic field produced by an infinite wire bearing constant current as defined by Equation (5) with $K = 0.01024 \text{Tm}$, and a rotational field given by Equation (7) with $C = 1.72 \cdot 10^{-10} \text{Tm}^4$.

Irrotational magnetic field: Figure 9 shows the velocity distribution in the vicinity of the stenosis. The top one corresponds to the case where there is no magnetic field, in the middle one the magnetization force is considered, and the last one shows the case where the magnetic field is on and both forces act on the biofluid. As expected, the irrotational magnetization force does not alter the velocity field. This is also true in the case where the Lorentz force is present. Due to the moderate electric conductivity of blood, the generated electric current density is not strong enough to alter the flow significantly. Figure 10 demonstrates that the pressure in the presence of the prescribed magnetic field adjusts in order to accommodate the magnetic pressure. The total pressure is equal to that when the magnetic field is turned off. Figure 11 presents the components of the velocity field along the axis of symmetry and the pressure. Specifically, Figure 11a demonstrates the relative difference for two cases: the streamwise velocity when only the magnetization force is present, $u_M$, and when both forces are present, $u_{LM}$, with respect to their unperturbed counterparts, $u$. As it can be seen, the effect of the Lorentz force produces a negligible deviation. The remaining two components of the velocity are close to zero, as expected, and their values along the axis of symmetry are plotted in Figures 11b and 11c. Finally, Figure 11d presents the different components of pressure along the axis of symmetry.

---

1A significant deviation of the flow can only take place in the presence of extreme magnetic fields. To all intents and purposes thus the Lorentz force produced by an infinite wire bearing constant current can not alter the flow of blood unless the magnitude becomes prohibitively large.
Rotational magnetic field: The magnetization force produced in this case is not only rotational but also is characterized by strong spatial gradients. As a result, a considerable deviation of the flow can take place even in the case of moderate magnetic fields, as it can be seen in Figures 12, 13, and 14. Finally, Figure 15 compares all the relevant quantities along the axis of symmetry of the stenosis for the unperturbed case to the one with the rotational magnetic field turned on. The effect of the force on the flow is apparent.

4.3. Case IIb: pulsatile flow through stenosis

For the pulsatile flow simulations, a sinusoidal flow waveform with a steady flow offset to prevent bulk flow reversal as shown in Figure 16 was used. The inverse Womersley method was used to compute the velocity profile prescribed at the inlet of the computational domain. Assuming a specific volumetric flow rate, \( Q(t) \), at the inlet it is possible to obtain the corresponding fully developed velocity profile, \( u(r/R,t) \), which, for a single harmonic, can be obtained from the following expression [31],

\[
\begin{align*}
\frac{Q(t)}{\pi R^2} & = \frac{\alpha R}{\alpha R^2} \left[ \frac{J_0(\alpha R^2) - J_1(\alpha R^2)}{J_0(\alpha R^2) - J_1(\alpha R^2)} \right], \\
\end{align*}
\]

where \( J_0 \) and \( J_1 \) are the Bessel functions of the zero and first kind, respectively. The dimensionless parameter \( \alpha \) is the Womersley parameter defined as,

\[
\alpha = R \left[ \frac{\sqrt{\frac{\mu}{\rho}}} \right],
\]

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Figure 11. Effects of the Lorentz and irrotational magnetization forces produced by a magnetic field defined by Equation (5) with $K = 0.01024$ Tm, along the axis of symmetry. The former does not alter the flow significantly due to the moderate electric conductivity of blood, and the latter manifests its presence as a pressure change only, leaving the kinematic part totally unaffected.

Figure 12. Streamwise velocity of the unperturbed flow (top) and in the presence of the magnetic field (7) with $C = 1.72 \cdot 10^{-10}$ Tm$^4$ (bottom).
which is a measure of the unsteadiness of the flow. Its value determines the relative strength of viscous to inertial forces. For values of $\alpha < 1$, the viscous forces dominate and the flow tracks the oscillating pressure gradient, yielding parabolic velocity profiles. For values of $\alpha > 1$, the flow is shifted relative to the pressure gradient, flattening the velocity profiles.

The computational domain extends 15 diameters downstream of the stenosis throat to allow sufficient post-stenotic flow development. The properties of the biofluid and the mesh are the same as those used in the steady flow simulations. The diameter at the inlet of the stenosis was set equal to 0.01 m, which fixes the values of the parameters $A$, $B$, and $x_0$ as pointed out in Section 2.2.2. Assuming a period of pulsation $T = 1.0$ s, the Womersley parameter $\alpha = 6.8$, with mean inlet $Re = 100$. Three flow cycles were computed with a time periodic solution reached at the third flow cycle from which results are presented.

Figures 17 and 18 present the streamwise velocity on the $xy$-plane spanning 1 diameter upstream and 10 downstream of the stenosis throat with the magnetic field defined by Equation (7) with $C = 1.72 \cdot 10^{-10}$ Tm$^4$ turned off and on, respectively. Results are presented at early systole ($t/T = 0$), peak systole ($t/T = 0.25$), mid-deceleration phase ($t/T = 0.5$), and end-deceleration phase ($t/T = 0.75$). Because of the strong spatial gradients of the magnetic field, the flow is significantly altered creating a considerable asymmetry in the size of the post stenotic recirculation regions.

Figure 19 presents the axial centerline velocity at the specified reference times with the magnetic field (7) switched on or off. In the former case, it is evident that the flow that rapidly accelerates through the throat of the stenosis obtains a maximum centerline velocity gradient at peak systole. At this point in time, the upstream centerline velocity is 30% higher than the mean inlet centerline velocity, $u_{avg}$, whereas the peak velocity at the throat is approximately 5.5 times greater than $u_{avg}$. The jet formed during systolic acceleration extends downstream of the stenosis throat as clearly shown by the mid-deceleration and end-deceleration curves. When the rotational magnetic field is
**Figure 15.** Effects of the magnetization force produced by the magnetic field defined by Equation (7) with $C = 1.72 \cdot 10^{-10}$ Tm$^4$. Neither the field nor the force are irrotational yielding a deviation of the flow even for moderate magnetic field strengths. The solid lines represent the unperturbed quantities and the dash lines the corresponding quantities when the field is turned on.

switched on, the velocity field is strongly altered, breaking the symmetry of the velocity profile. As a result, the peak velocity no longer lies at the centerline. The biofluid near the source of the field is drawn towards it. However, further downstream as the force diminishes the flow starts to oscillate slightly in the cross streamwise direction until it recovers its unperturbed state.

Figure 20 presents the time course of the axial wall shear stress at two points on opposite walls located on the $xy$-plane 5 diameters downstream of the stenosis throat. With the field switched
on flow symmetry is preserved. Flow separation at the interrogated location occurs during mid-acceleration and continues until early acceleration when the flow reattaches. When the field is switched on, flow symmetry is clearly broken with the wall location closer to the source of the magnetic field displaying only a very short period of flow detachment within the cycle. In contrast, the opposite wall region that lies further away from the magnetic field source is exposed to detached flow during most part of the flow cycle. It is also of note that the upper wall is exposed to a peak axial wall shear stress 12% higher when the field is switched on compared to that with field switched off.

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Figure 19. Effects of the magnetization force produced by the magnetic field $\mathbf{B}$ with $C = 1.72 \times 10^{-10}$ Tm$^4$ on an idealized pulsatile flow. Neither the field nor the force are irrotational yielding a deviation of the flow even for moderate field strengths. The solid lines represent the unperturbed axial centerline velocities at different time moments and the dash lines the corresponding velocities when the field is turned on.

Figure 20. Wall shear stress (WSS) at $x/D = 5$, $y/D = \pm 0.5$ in the two distinct cases when the magnetic field $\mathbf{B}$ with $C = 1.72 \times 10^{-10}$ Tm$^4$ is turned off and on. In the first case, the WSS is identical for the two symmetrical points on the domain (circles and squares), but when the field is acting on the biofluid the symmetry is broken (diamonds and triangles).
5. CONCLUSIONS

We present a mathematical and numerical description for the flow of an incompressible, Newtonian biomagnetic fluid between two parallel plates and through a straight tube with a 60% in diameter (84% on area) axisymmetric stenosis, in the presence of two external magnetic fields: (i) a non-uniform, irrotational field produced by an infinite wire bearing constant current; and (ii) a rotational magnetic field resembling an ideal dipole. In order to study the phenomenon in its generality, both the Lorentz and magnetization forces are included in the model. The generated electric current is calculated using Ohm’s law for electrically conducting moving fluids. Our numerical approach is based on a pressure-correction scheme for the incompressible Navier-Stokes equations which combines a DG approximation for the velocity and a CG approximation for the pressure.

We find that a time invariant and irrotational magnetic field produces a magnetization force that is also irrotational and does not alter the velocity field. It is the Lorentz force that can affect the kinematics, but an appreciable influence to the flow can only take place with extremely strong magnetic fields. The presence of the magnetization force, however, introduces an additional pressure-like term, the magnetic pressure. It is shown that the ordinary pressure is redistributed inside the fluid in such a way that the total pressure is identical to that obtained in the absence of the external magnetic field. Our numerical results are also supported by an analysis that shows that the effects of irrotational forces in fluid motion, under certain conditions, do not affect the velocity field.

However, in general, magnetic fields could generate rotational magnetization forces. In this case, an extra term in the RHS of the momentum equation appears. We have shown in the idealised stenosis model that, for both steady and time varying flow, even a moderate intensity rotational magnetic field with steep spatial gradients producing a rotational magnetization force can be used to alter the flow field of a biofluid. These findings are expected to have important applications in blood flow control.

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